Fermi National Accelerator Laboratory

FERMILAB-75/70-THY
September 1975

Heavy Quarks and Their Experimental Consequences*

THOMAS APPELQUIST‡
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

ABSTRACT

Recent theoretical work on heavy quark dynamics is reviewed. In the context of a color gauge theory of strong interactions, the structure of heavy quark-antiquark bound states and their decay properties is discussed. The emphasis of the talk is on the dynamical differences between heavy and light quark bound states. It is suggested that the former will more directly reflect the structure of the underlying field theory.

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*Invited talk presented at the Argonne Conference on New Directions in Hadron Spectroscopy. Argonne National Laboratory, July 9, 1975.

‡Alfred P. Sloan Foundation Fellow. On leave from Yale University.

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

INTRODUCTION

There have been several talks at this conference devoted to experimental work on the new high-mass narrow resonances. These talks, along with recent published results[1] have presented mounting evidence that the newly discovered particles are $J^P = 1^-$ hadrons. In this talk, I will assume this to be true and discuss the exciting possibility that they fit naturally into a quark constituent model of hadrons extended to include one or more heavy "charmed" quarks. If this is true, then the new particles may provide us with an important new experimental handle on the strong interactions. By this I mean that the features of the low lying states may reflect the properties of the underlying field theory more directly than the other hadrons. This is due to the large mass of the new quarks and their subsequent non-relativistic motion when bound together. I will first describe the colored quark gluon strong interaction model and then discuss the behavior of the e^+e^- total hadronic cross section. I will describe how perturbation theory is used to calculate the decay widths of the new particles.[2] Finally, I will review some of the phenomenological spectroscopic work and discuss its possible connection to the underlying field theory via the Bethe-Salpeter equation.

THE MODEL

I will assume that the strong interactions are described by a local, renormalization quantum field theory of quarks and vector gluons:

$$\mathcal{L} = -\tfrac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \bar{\psi}(i\not{D} - m_0)\psi \quad . \tag{1}$$

ψ is a set of quark fields coming in different flavors, u, d, and s along with one or more heavy quarks. Each flavor comes in three colors[3] of the readers choice and color is taken to be an exact SU(3) gauge symmetry. Thus each quark color multiplet has a single mass and the colored vector mesons remain massless. $F^{a}_{\mu\nu}$ is the gauge covariant curl and D_μ is the covariant derivative:

$$(D_\mu \psi)_n = \partial_\mu \psi_n - \tfrac{1}{2} g A^{a}_{\mu} (\lambda^{a})_{nm} \psi_m \quad , \tag{2}$$

where ψ_m is one of the color triplets. The symmetries of the theory (the breaking of $U(N) \times U(N)$ where N is the number of flavors) are just the symmetries of the bare mass matrix m_0. Each flavor has its own electrical charge and the colored gluons are electrically neutral.

For $N < 16\frac{1}{2}$, this model is asymptotically free.[4] This means that the short distance behavior of the theory is calculable in terms of an effective coupling constant $\bar{g}(M)$.[5] As $M \to \infty$, $\bar{g}^2(M) \sim 1/\log M$ and if Bjorken scaling is to be explained by this kind of theory, then for $M >$ 1 or 2 GeV, $\alpha_s(M) \equiv \frac{\bar{g}^2(M)}{4\pi} \ll 1$.[6] The asymptotic freedom of the theory is crucial and underlies most of the dynamical considerations in my talk. One of the central tenets of the colored quark dogma is that the physical hadron states should all be color singlets.[7] The absolute confinement of the colored quarks and gluons with an unbroken color gauge invariance is an attractive and widely discussed possibility and it could well be a consequence of the infrared instability of the asymptotically free gauge theory. Some quantitative support for this idea has come from the lattice gauge work of Wilson[8] and Kogut and Susskind[9] but the way it comes about, if at all, in a continuum theory is far from clear.[10]

My main concern will be with the strong interaction dynamics of the heavy quarks and my remarks should apply equally well to any model. Some of the discussion will assume the existence of a single heavy quark (c) and is appropriate for the original SU(4) model of Glashow, Illiopoulos and Maiani.[11] It can easily be modified to apply to more elaborate charmed quark models.[12] There are many ongoing experiments which bear directly on this question of the correct model.[1]

It is very important to discuss the mass renormalization of this model and particular to state precisely what is meant by the new quarks being heavier than the u, d and s quarks. If the quarks are permanently confined, this becomes a question requiring some thought. Heaviness is straightforward to define in terms of the bare masses in the large cutoff limit or equivalently in terms of renormalized masses defined at some mass scale $M > 1$ or 2 GeV. This might be called short distance heaviness. Each of the bare masses m_{oi}, $i = u, d, s, \ldots$ is logarithmically divergent in perturbation theory. Since the origin is an ultraviolet stable fixed point, the factors of $(\log \Lambda)^n$ can be summed to produce the behavior $m_{oi}(\Lambda) \sim (\log \Gamma)^{-\epsilon}$ where ϵ is a positive anomalous dimension, calculable from one loop diagrams, and independent of flavor.[13] Thus $m_{oi}(\Lambda) \to 0$ as $\Lambda \to \infty$, and the ratios $m_{oi}(\Lambda)/m_{oj}(\Lambda)$ are finite. The statement that a new quark labeled by c is heavy in the short distance sense is just the statement that for $i = u, d, s$,

$$\lim_{\Lambda \to \infty} \frac{m_{oc}(\Lambda)}{m_{oi}(\Lambda)} \gg 1 \quad .$$

This property can be equivalently stated in terms of renormalized masses $m_i(m)$ [14] defined at an energy scale M .

An additional prescription must be introduced having to do with the overall mass scale to arrive at a definition of heaviness that is useful for the calculations I will describe. A renormalized mass matrix m_i is obtained by a rescaling of the bare mass matrix, $m_i = Z\, m_{oi}$. Then the m_i have exactly the same ratios as the $m_{oi}(\Lambda)$ in the $\Lambda \to \infty$ limit. It is convenient to adjust Z so that for the heavy quark c (assuming there is only one), m_c is the threshold of a cut to any finite order of perturbation theory. This imitates quantum electrodynamics but unlike that theory, the threshold behavior to all orders is unknown due to infrared instability. The additional heaviness assumption is then $m_c >$ 1 or 2 GeV , i.e., that $\alpha_s(m_c) << 1$. This is crucial for the perturbative calculations I will describe and in fact means that the parameter m_c is in principle experimentally accessible.

$\sigma_{tot}(e^+e^- \rightarrow \text{HADRONS})$

The new resonances were simultaneously discovered in a hadron induced reaction[15] and in e^+e^- annihilation.[16] A discussion of the annihilation process is easier theoretically and serves as an introduction to the other parts of my talk. It is intended only as a brief summary of the corresponding discussion in Ref. 2.

The object of most direct theoretical concern is the total cross section which is conveniently normalized by the total $\mu^+\mu^-$ cross section:

$$R(E_{CM}^2) = \sigma_{tot}(E_{CM}^2) \Big/ \frac{4\pi\alpha^2}{3E_{CM}^2} \quad . \tag{3}$$

I will summarize the expected behavior of $R(E_{CM}^2)$ assuming there is only one heavy quark with mass m_c as defined above.

1. The most direct and reliable use of asymptotic freedom is the calculation of Euclidean Green's functions. The hadronic vacuum poliarization $\Pi(q^2)$ is such an object and for $-q^2 > 1$ or $2\ \text{GeV}^2$, it can be calculated in perturbation theory. This leads, through a dispersion relation, to a bound on the integrated total cross section.[17] In addition to this, Healy[18] has been able to place some further weak but rigorous bounds on the behavior of the total cross section. These follow from

analyticity and the spacelike behavior given by asymptotic freedom.

2. Some justification can be given to the use of perturbation theory for the direct calculation of $R(E_{CM}^2)$ at high energies. First suppose there is no c quark. For $E_{CM} > 1$ or 2 GeV (well above m_u, m_d and m_s), perturbation theory converges in low orders since $\alpha_s(M)$ is small for $M > 1$ or 2 GeV. The absence of large logarithms which could destroy convergence is guaranteed by the mass singularity theorem of Kinoshita[19] which says that the limit $m_u, m_d, m_s \to 0$ exists for any diagram contributing to $R(E_{CM}^2)$. The problem with this argument is that at any finite value of E_{CM}, one is always passing through multiple quark thresholds. In perturbation theory, these will appear in high orders and then the existence of the $m_{u,d,s} \to 0$ limit is not relevant. New small momenta (the distance to nearby thresholds) can enter the problem and the expansion may break down. In Ref. 2, some arguments are given that these high order effects average out to a small contribution to $R(E_{CM}^2)$. If this is true, then one arrives at the quark-parton prediction

$$R(E_{CM}^2) = \sum_i Q_i^{\,2}\{1 + 0(\alpha_s)\} \tag{4}$$

with $\sum_i Q_i^{\,2} = 2$. It can be shown rigorously[20] that this result holds for R averaged over intervals on the order of 1 GeV. It remains a challenge to completely deal with the multiple thresholds and understand why experiment (which averages over ~ 1 MeV intervals) is given rather well by (4) below 3 GeV.

3. With the light quarks assumed to produce a constant background, the effect of the heavy c quark can be analyzed in perturbation theory. Since $E_{CM} > 1$ or 2 GeV, the perturbation expansion will only break down in low orders if some other small dimensional parameter enters the computation. Near the two heavy quark threshold, this is provided by the difference $E_{CM} - 2\,m_c$ or equivalently the average momentum transfer in the $c\bar{c}$ bound state. Thus outside of a region of about 2 GeV in width centered on $2\,m_c$, perturbation theory should be reliable.

On the low side, the heavy quark will not contribute and the light quark parton model should be reliable. On the high side, perturbation theory can be used to predict the approach to asymptotic scaling ($R \to 10/3$ in the SU(4) model). This is discussed at length in Ref. 2 and I will just remind you that the rate of approach (from above) depends on the parameter m_c in a calculable way. Thus a measurement of the approach is a measurement of m_c, a mass parameter of a possibly confined heavy quark.

It is by now very unlikely that the single heavy quark of the SU(4) model can explain the total cross section behavior since R seems to be staying above five through $E_{CM} \approx 7$ GeV. Nevertheless, the ideas discussed here should still be relevant once the correct flavor and/or heavy lepton theory is determined and the cross section measured through $E_{CM} \approx 9$ GeV.

4. For the transitional region the perturbation expansion breaks down in two ways. First of all, the exchange of massless gluons leads, in the absence of Yang-Mills radiative connections, to Coulombic singularities just as in QED. These factors of $1/\left(E_{CM}^2 - 4\, m_c^2\right)^n$ produce a Balmer series of resonances accumulating at threshold. The widths can be calculated since the dominant decay mode is into three gluons. That this picture cannot be correct and that the Yang-Mills structure must be important, can be seen by estimating the radius of the Coulombic bound states

$$\langle r\rangle_n = \frac{2}{4/3\alpha_s} \frac{1}{m_c} n^2 \quad . \tag{5}$$

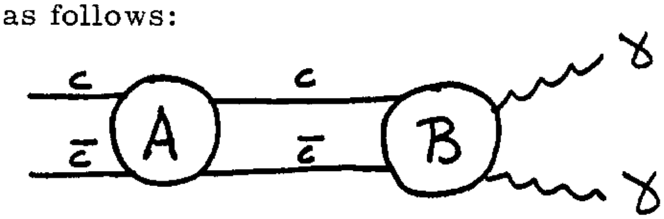
For $\alpha_s \leq 0.3$ and $m_c \leq 2$ GeV , $\langle r\rangle_n \geq \frac{1}{200 \text{ MeV}} n^2$. Thus even the ground state is probably too large (with typical momentum transfers ≤ 200 MeV) to believe the Coulombic picture. This is also shown by the phenomenological work on $c\bar{c}$ spectroscopy which I will briefly discuss.

The importance of the Yang-Mills corrections is signaled by the presence of large logarithms $\log (\langle k\rangle / M)$ where $\langle k\rangle$ is the typical momentum transfer. This is the second kind of perturbation theory breakdown and means that the properties of the bound states are not Coulombic, there is now good evidence that they are non-relativistic, weakly bound systems. This picture has emerged from the phenomenological work of several groups and I will return to it in the last part of my talk. First of all, I will discuss how this fact allows one to calculate the ratios of decay widths even though perturbation theory for the binding has broken down.

DECAY WIDTHS IN PERTURBATION THEORY

The binding strength is conveniently measured against the mass parameter m_c, as defined above. If the typical momentum transfers are small relative to m_c or equivalently if the bound state mass M_B is such that $2\,m_c - M_B << M_B$, then the system is weakly bound. This seems to be the case for the ψ and ψ', meaning that they are essentially two heavy-quark systems (no additional $c\bar{c}$ pairs). The single $c\bar{c}$ pair can be thought of as moving in a static potential produced by the gluons and light quarks.

Consider first the electromagnetic decays of the ψ and its predicted pseudoscalar partner, the η_c. In the case of the η_c decay, this involves a transition from a $c\bar{c}$ state to a two photon state. It can be pictured as follows:



The B amplitude is defined to be two particle irreducible in the decay channel and everything else is grouped into the A amplitude. Suppose I try to calculate both of them as a perturbation expansion in $\alpha_s(M)$. The A amplitude contains both Coulombic and Yang-Mills singularities near threshold and the perturbation expansion collapses. To zeroth order in the binding, the $c\bar{c}$ pair must come together to annihilate and the expression for the decay width is

$$\Gamma(\eta_c \to \gamma\gamma) = |\psi(0)|^2 \, |M_{\gamma\gamma}|^2 \quad .$$

The wave function depends on the A amplitude and cannot be calculated in perturbation theory.

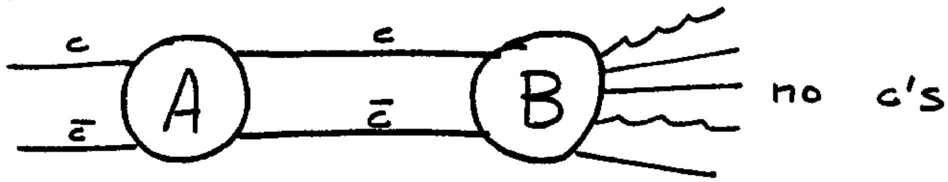
The matrix element M is calculated from the B amplitude at threshold. We have analyzed this amplitude through several orders in α_s in a model without light quarks.[21] We find that because of its $c\bar{c}$ irreducible structure, there are no singularities that would cause a breakdown of perturbation theory. Note that this crucially depends on m_c being large; the only small dimensional parameters are $p_i^2 - m_c^2$ and $M_{\eta_c} - 2m_c$ and the B amplitude is insensitive to this. Thus B is dominated by the Born graphs and

$$\Gamma(\eta_c \to \gamma\gamma) = |\psi(0)|^2 \frac{16}{3} (Q_c\alpha)^2 \frac{1}{m_c^2} \quad , \tag{6}$$

where $Q_c\alpha$ is the charge of the c quark. The value of the analysis leading to Eq. 6 is that the same wave function factor $\psi(0)$ enters several decay widths. Thus if the B amplitudes can be calculated, ratios of decay widths can be predicted. For example, $\mu^+\mu^-$ decay of the ψ is given by[2,22]

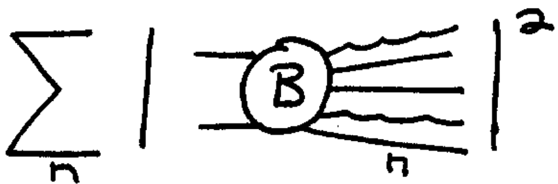
$$\Gamma(\psi \to \mu^+\mu^-) = \frac{3}{4}\,\Gamma(\eta_c \to \gamma\gamma) \quad . \tag{7}$$

The next, and most interesting, step is to extend this analysis to the hadronic decays of the ψ and η_c. In terms of quarks and gluons, this involves a transition from the $c\bar{c}$ state to states not containing heavy quarks.



The B amplitude is again defined to be two particle irreducible in any order. It contains any number of light quarks and gluons in the final state depending on the order of perturbation theory.

To calculate the total transition probability to any order, one must square and sum over final states



This object has been analyzed through low orders in α_s. It is found that because of $c\bar{c}$ irreducibility and because of the sum over all real and virtual processes to each order in α_s, there are no singularities. It is finite in the limit $p_i^2 \to m_c^2$, $E_{CM} \to 2m_c$ and in the limit m_u, m_d, $m_s \to 0$. Note that unlike the electromagnetic decay, B itself is not finite in these limits; only $|B|^2$ summed over final states.[23] This is similar to the calculation of σ_{tot} and as in that case, it is related to the Kinoshita mass singularity theorem.[19]

There is thus no obvious source for the breakdown of perturbation theory so that the dominant term should be the Born term. In the case of $\eta_c(\psi)$, the Born term involves a transition to the two (three) gluon final state. This use of the minimal gluon state to calculate the total hadronic width is similar to the use of the quark-parton model to calculate $\sigma_{tot}(e^+e^- \to \text{hadrons})$. In each case, the mechanism by which the underlying quanta evolve into physical hadrons remains unclear but the perturbation theory analysis suggests that this will not affect the total transition rate. The decay width of the ψ is[2]

$$\Gamma(\psi \to \text{hadrons}) = |\psi(0)|^2 \frac{16}{9\pi}(\pi^2-9)\frac{5}{18}\alpha_s^3 \frac{1}{m_c^2} \tag{8}$$

and the narrow width of the ψ follows from an $\alpha_s(M)$ of 0.2 at $M \approx 2$ GeV. Numerical estimates of this and other decays are included in Ref. 2.

The hadronic width of the η_c can now be estimated. Since it can communicate with ordinary hadrons through only two gluons, it is predicated to be much broader than the ψ. We find[2] $\Gamma(\eta_c \to \text{hadrons}) \approx$ 5 MeV, a factor of 100 broader than the ψ. I want to emphasize that this prediction is the crucial test of the ideas presented here. This is true even if the ultimate explanation for the narrowness of the new particles is more complicated than simple perturbation theory. Even if the perturbation analysis is misleading and higher orders are important, the lowest order contribution should still be a component of the whole story. A very broad η_c would be a welcome piece of support for the colored quark gluon theory.

Weak binding is essential. Binding corrections for η_c and ψ decay have been discussed in Ref. 3 and by Fritzsch and Minkowski[24] for a range of values of m . For light hadrons, analogous decays (for example, $\phi \to 3\pi$) are much more difficult to estimate. Since $1/m_s$ is likely on the order of the ϕ radius (≈ 1 fermi), the local annihilation mechanism fails completely. Asymptotic freedom might still be playing a role in the smallness of this width however.[2,24,25]

The minimal gluon mechanism is based on the observation that to any order, there is no sensitivity in B to small dimensional parameters ($M_\psi - 2m_c$, $m_{u,d,s}$, $m_{gluon} = 0$). Just as in σ_{tot}, the presence of higher light quark thresholds in the vicinity of M_ψ must be kept in mind. There is another possible source of a small dimensional parameter which comes to mind. If many gluons are emitted, then perhaps some small fraction of the total mass, corresponding to a multigluon division of the momentum, enters in higher orders producing large logarithms. The analysis of whether this happens is very difficult. However, if it does happen here then it probably happens in the calculation of σ_{tot} or even Euclidean Green's functions. The entire use of asymptotic freedom might then be in question.

$c\bar{c}$ SPECTROSCOPY AND THE BETHE-SALPETER EQUATION

Several groups[25,26,27,28] have examined the spectrum of $c\bar{c}$ states. They have all chosen to respresent the long range binding by a linearly rising coordinate space potential. Two important things have emerged from this work. First of all, there are experimental predictions for masses and widths of additional $c\bar{c}$ states. Secondly, there is a qualitative feature which is very likely independent of the particular form of the potential. This is the non-relativistic, loosely bound structure which I have exploited in the decay rate calculations. I now want to discuss the importance of this feature in understanding the dynamics of $c\bar{c}$ binding.

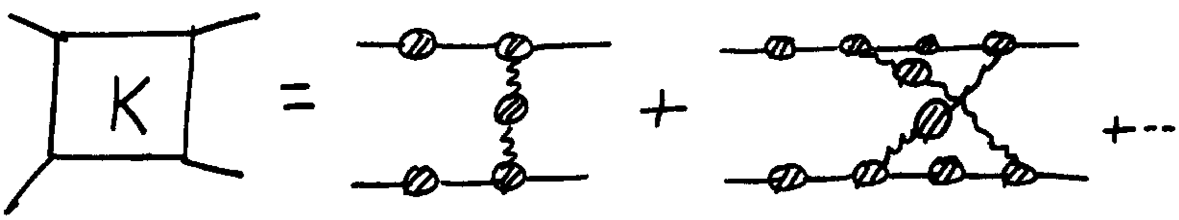
The potential used to analyze the $c\bar{c}$ spectrum in Ref. 27 is

$$V(r) = V_0 = \frac{\alpha_s}{r} + a\, r \quad . \tag{9}$$

A Schrödinger equation is solved with the mass m_c for the quark. The constant V_0 represents intermediate range forces so that no attempt is made to calculate the absolute value of the $c\bar{c}$ mass scale in terms of m_c and a. A fit to the ψ and ψ' masses and the leptonic width of the ψ fixes the parameters. It is found that the Coulombic term α_s/r is not important and that $a \ll m_c^2$. The classical velocity in the bound state is proportional to $(a/m_c^2)^{1/3}$ [26,29] and the ψ and ψ' are found to be nonrelativistic. This is a consistency check on the use of a spin independent Schrödinger equation and means that relativistic and spin dependent effects can be treated perturbatively.

Many rough experimental predictions can be made before worrying about fine structure. P-wave states are predicted to exist at about 3.5 GeV and the ψ and ψ' should be accompanied by $J^{PC} = 0^{-+}$ states. Radiative transition widths between these states can be computed[27, 29, 30] and confronted with experiment. For example, the transition width from ψ' to the η_c partner of the ψ is predicted to be on the order of 1 KeV. This is well below the current upper bound of about 10 KeV for photons in the 600 MeV energy region.[31] Transitions from the ψ' to the P-wave states should occur with widths up to about 100 KeV if these states do indeed sit at about 3.5 GeV. The current upper bound on the width for photons of these energies has been quoted at around 20 KeV.[31] This may be a problem and it is very important to test the sensitivity of the predictions to the details of the model.

If the low lying $c\bar{c}$ states are nonrelativistic, then unlike the other hadrons, the Bethe-Salpeter equation might be a useful formalism.[33] The Bethe-Salpeter kernel can be skeleton expanded in terms of dressed propagators and vertices



It is natural to conjecture that since the system is nonrelativistic, the expansion can be truncated at the single dressed ladder level. This is suggested by QED where the single (undressed) ladder gives the Schrödinger equation along with the lowest order hyperfine splitting and some other relativistic corrections.

Let me first suppose this is true and consider the structure of the dressed ladder. The relevant kinematic limit is $p_i^2 - m_c^2 << m_c^2$ for $i = 1, \ldots 4$ and $q^2 << m_c^2$. In this limit, the important graphs will be those without c quark internal loops.[32] Essentially all such vertex and self energy corrections are important because they contain logarithmic singularities as $p_i^2 \to m_c^2$ or $q^2 \to 0$. These large logarithms can compensate the factors of α_s. In this mass shell limit, the vertex has a convective piece proportional to $\frac{p_\mu + p_\mu'}{2m_c}$ and a spin dependent piece proportional to $\sigma_{\mu\nu} q^\nu / m_c$. With external propagator corrections included, each piece comes with a form factor singular in the limit $p_i^2 \to m_c^2$ and $q^2 \to 0$.

The first piece produces the long range spin independent potential. The spin dependent form factor has a structure similar to the spin independent one in perturbation theory although the structure to all orders is not clear. As an extreme example, suppose that the only important piece of the vertex is proportional to γ_μ. This would lead to identical spin independent and spin dependent form factors, and just as in QED, the spin dependent potential would be given by the Laplacian of the spin independent potential. For a spin independent potential of the form $V(r) = a\,r$, the s-wave spin-spin interaction is found to be $1/3\ a/m_c \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{r}$.[29,34]

This leads to a $\psi - \eta_c$ splitting of about 90 MeV.[29]

The question of whether the higher terms in the kernel expansion are suppressed is being examined.[33] The answer is tentatively yes. For example, in the linear potential model $V(r) = a\,r$, the crossed ladder is suppressed by a factor of $a/m_c^{\,2}$ relative to the single uncrossed ladder.

For relativistic systems, the Bethe-Salpeter equation could not be useful in the above way since the kernel contributions could not be delineated. Several groups[25,28] have assumed that the linear potential is inherent to all systems and used it with a relativistic wave equation to treat the light hadrons along with the $c\bar{c}$ system. These efforts have met with interesting numerical success but the above arguments make me very uneasy about this approach. The difference between a relativistic and nonrelativistic system is a dynamical difference, not just kinematical. If for example the $c\bar{c}$ potential comes from the single dressed ladder term in the kernel, it would be very surprising if this same potential had anything to do with light quark systems.

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[5]For a discussion of the renormalization of the theory and the precise definition of $\bar{g}(M)$, see the second paper of Ref. 3.

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[22]In view of the breakdown mentioned in Ref. 21, this is the weakest of the predictions discussed here.

[23]Here again, we have not proven this to all orders. Such a proof should be possible as a variation on the proof of Kinoshita (Ref. 19).

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